Università degli Studi di Ferrara
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Costanti Secondarie κ e ζ

Appunti di Campi Elettromagnetici
di
Tarin Gamberini

Corso di Campi Elettromagnetici (ante riforma 3+2)
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Indice

Introduzione

In questo breve articolo si calcolano le espressioni generali delle costanti secondarie κ e ζ relative ad un onda piana che si propaga in un mezzo dissipativo e non dispersivo.

Sono possibili due strade: la prima è adottata dal libro di testo[1] e separa le espressioni complesse di κ e ζ nelle due parti reale ed immaginaria; la seconda, che ho svolto per esercizio e riportato di seguito, lavora direttamente sulle espressioni complesse.

Tali espressioni generali possono essere poi *specializzate* al caso in cui il mezzo attraverso il quale si propaga l'onda sia un *buon* conduttore oppure un *buon* dielettrico.

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[1] Lezioni di Campi Elettromagnetici - Gerosa Lampariello - McGraw Hill

Capitolo 1

Costanti Secondarie

1.1 Definizione delle costanti secondarie

Consideriamo un mezzo dissipativo ($\sigma \neq 0$) e non dispersivo ($\epsilon = \epsilon_R - j\epsilon_J$ con $\epsilon_J = 0$), possiamo definire:

$$\epsilon_c = \epsilon - j\frac{\sigma}{\omega} \tag{1.1.1}$$

Consideriamo un sistema di riferimento ortogonale tale per cui l'onda piana risulti propagarsi lungo una direzione individuata da una sola componente. In questo modo il vettore complesso $\underline{\kappa} = \underline{\beta} - j\underline{\alpha}$ risulta essere uno scalare complesso $\kappa = \beta - j\alpha$.

La definizione delle costanti secondarie può essere data sotto forma di numero complesso qualora si espliciti ϵ_c:

$$\kappa = \omega\sqrt{\mu\epsilon_c} = \beta - j\alpha \tag{1.1.2}$$

$$\zeta = \sqrt{\frac{\mu}{\epsilon_c}} = \zeta_R + j\zeta_J \tag{1.1.3}$$

Calcoliamo le parti reali e complesse di κ sostituendo la 1.1.1 nella 1.1.2, otteniamo:

$$\kappa = \omega\sqrt{\mu\left(\epsilon - j\frac{\sigma}{\omega}\right)} = \omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\epsilon\omega}} \tag{1.1.4}$$

La radice del numero complesso[1] si calcola agevolmente passando alla rappresentazione in modulo e fase, ossia detti:

$$M = \sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} \qquad \Phi = -\arctan\frac{\sigma}{\epsilon\omega} \tag{1.1.5}$$

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[1] Vedere appendice A.

riscriviamo la 1.1.4 come:

$$\kappa = \omega\sqrt{\mu\epsilon}\sqrt{Me^{j\Phi}} = \omega\sqrt{\mu\epsilon M}e^{j\frac{\Phi}{2}} = \tag{1.1.6}$$

$$= \omega\sqrt{\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}e^{-j\frac{1}{2}\arctan\frac{\sigma}{\epsilon\omega}} \tag{1.1.7}$$

infine sviluppando l'esponenziale complesso:

$$\boxed{\beta = \Re[\kappa] = \omega\sqrt{\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}\cos\left(\frac{1}{2}\arctan\frac{\sigma}{\epsilon\omega}\right)} \tag{1.1.8}$$

$$\boxed{\alpha = \Im[\kappa] = \omega\sqrt{\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}\sin\left(\frac{1}{2}\arctan\frac{\sigma}{\epsilon\omega}\right)} \tag{1.1.9}$$

Calcoliamo le parti reali e complesse di ζ sostituendo la 1.1.1 nella 1.1.3, otteniamo:

$$\zeta = \sqrt{\frac{\mu}{\epsilon - j\frac{\sigma}{\omega}}} = \sqrt{\frac{\mu}{\epsilon}\frac{1}{1-j\frac{\sigma}{\epsilon\omega}}} \tag{1.1.10}$$

Detti M e Φ come nelle 1.1.5 riscriviamo la 1.1.10 come:

$$\zeta = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\sqrt{Me^{j\Phi}}} = \sqrt{\frac{\mu}{\epsilon M}}\frac{1}{e^{j\frac{\Phi}{2}}} \tag{1.1.11}$$

$$= \sqrt{\frac{\mu}{\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}}e^{j\frac{1}{2}\arctan\frac{\sigma}{\epsilon\omega}} \tag{1.1.12}$$

infine sviluppando l'esponenziale complesso:

$$\zeta_R \boxed{= \Re[\zeta] = \sqrt{\frac{\mu}{\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}}\cos\left(\frac{1}{2}\arctan\frac{\sigma}{\epsilon\omega}\right)} \tag{1.1.13}$$

$$\zeta_J \boxed{= \Im[\zeta] = \sqrt{\frac{\mu}{\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}}\sin\left(\frac{1}{2}\arctan\frac{\sigma}{\epsilon\omega}\right)} \tag{1.1.14}$$

1.2 Costanti κ e ζ per un buon conduttore

Per buon conduttore intendiamo un mezzo in cui sia valida l'approssimazione:

$$\sigma >> \epsilon\omega \qquad \frac{\sigma}{\epsilon\omega} >> 1 \qquad \frac{\sigma_c}{\epsilon\omega} >> 1 \tag{1.2.1}$$

in cui con σ_c indichiamo la conducibilità del conduttore.

Introdotta tale approssimazione nelle 1.1.8 e 1.1.9 otteniamo:

$$\kappa = \beta - j\alpha \approx \omega\sqrt{\mu\epsilon\sqrt{\left(\frac{\sigma_c}{\epsilon\omega}\right)^2}}\left(\cos\frac{1}{2}\frac{\pi}{2} - j\sin\frac{1}{2}\frac{\pi}{2}\right) = \tag{1.2.2}$$

$$= \omega\sqrt{\mu\epsilon\frac{\sigma_c}{\epsilon\omega}}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = \tag{1.2.3}$$

$$= \sqrt{\omega\mu\sigma_c}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = \tag{1.2.4}$$

$$= \sqrt{\frac{\omega\mu\sigma_c}{2}} - j\sqrt{\frac{\omega\mu\sigma_c}{2}} \tag{1.2.5}$$

Pertanto per un buon conduttore $\kappa = \beta - j\alpha$ è:

$$\boxed{\begin{aligned} \beta &\approx \sqrt{\frac{\omega\mu\sigma_c}{2}} && (1.2.6) \\ \alpha &\approx \sqrt{\frac{\omega\mu\sigma_c}{2}} && (1.2.7) \end{aligned}}$$

Introdotta tale approssimazione nelle 1.1.13 e 1.1.14 otteniamo:

$$\zeta = \zeta_R + j\zeta_J \approx \sqrt{\frac{\mu}{\epsilon\sqrt{\left(\frac{\sigma_c}{\epsilon\omega}\right)^2}}}\left(\cos\frac{1}{2}\frac{\pi}{2} + j\sin\frac{1}{2}\frac{\pi}{2}\right) = \tag{1.2.8}$$

$$= \sqrt{\frac{\mu}{\epsilon\frac{\sigma_c}{\epsilon\omega}}}\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = \tag{1.2.9}$$

$$= \sqrt{\frac{\omega\mu}{\sigma_c}}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = \tag{1.2.10}$$

$$= \sqrt{\frac{\omega\mu}{2\sigma_c}} + j\sqrt{\frac{\omega\mu}{2\sigma_c}} \tag{1.2.11}$$

Pertanto per un buon conduttore $\zeta = \zeta_R + j\zeta_J$ è:

$$\boxed{\begin{aligned} \zeta_R &\approx \sqrt{\frac{\omega\mu}{2\sigma_c}} && (1.2.12) \\ \zeta_J &\approx \sqrt{\frac{\omega\mu}{2\sigma_c}} && (1.2.13) \end{aligned}}$$

1.3 Costanti κ e ζ per un buon dielettrico

Per buon dielettrico intendiamo un mezzo in cui sia valida l'approssimazione:

$$\sigma << \epsilon\omega \qquad \frac{\sigma}{\epsilon\omega} << 1 \qquad \frac{\sigma_d}{\epsilon\omega} << 1 \tag{1.3.1}$$

in cui con σ_d indichiamo la conducibilità del dielettrico.

Introdotta tale approssimazione nelle 1.1.8 e 1.1.9 otteniamo:

$$\kappa = \beta - j\alpha \approx \omega\sqrt{\mu\epsilon\sqrt{1}}\left(\cos\frac{1}{2}\frac{\sigma_d}{\epsilon\omega} - j\sin\frac{1}{2}\frac{\sigma_d}{\epsilon\omega}\right) \approx \tag{1.3.2}$$

$$\approx \omega\sqrt{\mu\epsilon}\left(1 - j\frac{1}{2}\frac{\sigma_d}{\epsilon\omega}\right) = \tag{1.3.3}$$

$$= \omega\sqrt{\mu\epsilon} - j\omega\sqrt{\mu\epsilon}\frac{1}{2}\frac{\sigma_d}{\epsilon\omega} = \tag{1.3.4}$$

$$= \omega\sqrt{\mu\epsilon} - j\sqrt{\omega^2\mu\epsilon\frac{\sigma_d^2}{4\epsilon^2\omega^2}} = \tag{1.3.5}$$

$$= \omega\sqrt{\mu\epsilon} - j\frac{\sigma_d}{2}\sqrt{\frac{\mu}{\epsilon}} \tag{1.3.6}$$

Pertanto per un buon dielettrico $\kappa = \beta - j\alpha$ è:

$$\boxed{\beta \approx \omega\sqrt{\mu\epsilon}} \tag{1.3.7}$$

$$\boxed{\alpha \approx \frac{\sigma_d}{2}\sqrt{\frac{\mu}{\epsilon}}} \tag{1.3.8}$$

Introdotta tale approssimazione nelle 1.1.13 e 1.1.14 otteniamo:

$$\zeta = \zeta_R + j\zeta_J \approx \sqrt{\frac{\mu}{\epsilon\sqrt{1}}}\left(\cos\frac{1}{2}\frac{\sigma_d}{\epsilon\omega} + j\sin\frac{1}{2}\frac{\sigma_d}{\epsilon\omega}\right) \approx \tag{1.3.9}$$

$$\approx \sqrt{\frac{\mu}{\epsilon}}\left(1 + j\frac{1}{2}\frac{\sigma_d}{\epsilon\omega}\right) = \tag{1.3.10}$$

$$= \sqrt{\frac{\mu}{\epsilon}} + j\sqrt{\frac{\mu}{\epsilon}}\frac{1}{2}\frac{\sigma_d}{\epsilon\omega} \tag{1.3.11}$$

Pertanto per un buon dielettrico $\zeta = \zeta_R + j\zeta_J$ è:

$$\boxed{\zeta_R \approx \sqrt{\frac{\mu}{\epsilon}}} \tag{1.3.12}$$

$$\boxed{\zeta_J \approx \sqrt{\frac{\mu}{\epsilon}}\frac{\sigma_d}{2\epsilon\omega}} \tag{1.3.13}$$

Appendice A

Radice di un numero complesso

Ricordiamo brevemente che dato $z \in \mathbb{C}$, definiamo *radice n-ma* di z la funzione:

$$f_n(z) = \begin{cases} 0 & \text{se } z = 0, \\ \sqrt[n]{|z|}e^{j\frac{Arg\ z\ +\ 2\pi k}{n}} & \text{altrimenti,} \end{cases} \tag{A.0.1}$$

dove $k \in \mathbb{Z}$ e $Arg\ z \in \,]-\pi, \pi]$ è l'argomento principale di z.

Per i numeri complessi rappresentiamo la $f_n(z)$ col simbolo $\sqrt[n]{z}$, graficamente identico al corrispondente utilizzato con i reali ma concettualmente diverso.

Se $k = 0$ la $f_n(z)$ è detta *radice n-ma principale* di z. Per esempio la radice quadrata principale di z varrà:

$$\sqrt{z} = \begin{cases} 0 & \text{se } z = 0, \\ \sqrt{|z|}e^{j\frac{Arg\ z}{2}} & \text{altrimenti.} \end{cases} \tag{A.0.2}$$

Osserviamo infine che la *radice n-ma principale* di $z = z_R + j0$, con $z_R \in \mathbb{R}^+$, coincide con l'*usuale* radice n-ma di z_R.

Appendice B

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If you have Invariant Sections without Cover Texts, or some other combination of the three, merge those two alternatives to suit the situation.

If your document contains nontrivial examples of program code, we recommend releasing these examples in parallel under your choice of free software license, such as the GNU General Public License, to permit their use in free software.